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# Comment on "Solitons and excitations in the duality-based matrix model"

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Abstract: It is shown that a method for constructing exact multi-solitonic solutions of the coupled BPS equations in the duality-based generalization of the hermitean matrix model, which was put forward in a recent paper, is not correct.

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In the paper *JHEP* **08** (2005) 064 [1], Andrić et al. claim that a specific, duality-based generalization of the hermitean matrix model possesses topologically non-trivial, multi-solitonic solutions. In the BPS limit, they have constructed two-soliton and n-soliton solutions to the coupled non-linear equations. Unfortunately, the purpose of this Comment is to point out an error in their derivation.

Following ref. [1], the solutions to the coupled BPS equations (eqs. (2.4) and (2.5) in ref. [1]) are always interrelated by the constraint (eq. (2.7) in ref. [1])

$$\rho(x)m(x) = c, \tag{1}$$

where $c$ is some positive constant. This means that there is only one relevant equation since the other one is generated by the substitution $\rho(x) = \frac{c}{m(x)}$. The key point missed by Andrić et al. is the fact that the equivalence of the BPS equations was established on the basis of interchanging $\rho$ by $\frac{c}{m}$ (or vice versa), but only for $c \neq 0$.

Generally, for $c = 0$, the two equations are completely independent of each other, and must be simultaneously considered in order to get the right solutions. While the authors of ref. [1] checked that the one-soliton solutions $\rho(x)$ and $m(x)$ (eqs. (2.15) and (2.14) in ref. [1]) indeed satisfy both BPS equations in the $c \to 0$ limit, they failed to verify both field equations in the two-soliton case. Inserting expressions for $\rho(x)$ and $m(x)$ (eqs. (2.22) and (2.23) in ref. [1]) into the BPS system (eqs. (2.4) and (2.5) in ref. [1]), we find that the first equation is identically satisfied, whereas the second one is satisfied if

$$[\delta(x - x_0) - \delta(x + x_0)] \left( \frac{1-\lambda}{2x_0} + \lambda \fint \frac{dy\rho(y)}{x_0 - y} \right) = 0\,, \tag{2}$$

which implies that

$$\frac{1}{x_0^2 - b^2} + \frac{1}{x_0^2 - \bar{b}^2} = \frac{1}{2x_0^2}. \tag{3}$$

After introducing

$$b = |b|e^{i\varphi} \qquad \text{and} \qquad r = \frac{x_0^2}{|b|^2} \tag{4}$$

into the condition (3), we obtain

$$3r^2 - 2r\cos 2\varphi - 1 = 0. \tag{5}$$

Finally, this condition and the similar one (2.26) from ref. [1],

$$r - 2\cos\varphi + 1 = 0, \tag{6}$$

lead to

$$r^2 = 1 \qquad \text{and} \qquad \cos\varphi = 1, \tag{7}$$

which means that $x_0^2 = b^2$, and this brings us back to the constant solution $\rho = \rho_0$. However, because of the constraint

$$\rho(x)m(x) = 0, \tag{8}$$



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this is possible only for $\rho_0 = 0$ or $\lambda = 1$, which is of course unacceptable. Therefore, the proposed functions $\rho$ and $m$ (eqs. (2.22) and (2.23) in ref. [1]) are not the solutions of the BPS equations. Consequently, the $n$-soliton generalizations given by eq. (2.29) in ref. [1] are not the solutions of the BPS equations either.

Before concluding we would like to mention that one-soliton solutions studied in ref. [1] were known earlier in the context of the two-family Calogero model in the collective-field description (ref. [2]). The same references have already offered conceivable arguments against the existence of the two-solitons solutions of the form proposed in ref. [1].

Finally, we would like to notice that for $\lambda = 1/2$, i.e. in the so-called duality-based hermitean matrix model there are no vortex solitons at all. Namely, in this case, the soliton in eq. (2.15) reduces to the uniform density $\rho_0$.

In conclusion, the approach suggested in ref. [1] is not correct. The problem of obtaining closed-form multi-soliton solutions of the coupled BPS equations ((2.4) and (2.5) in ref. [1]) remains unsolved.


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## References


[1] I. Andrić, L. Jonke and D. Jurman, *Quantum fluctuations in a dual matrix model*, *JHEP* **08** (2005) 064 [hep-th/0411179].

[2] V. Bardek and S. Meljanac, *Solitons in the Calogero model for distinguishable particles*, *Europhys. Lett.* **72** (2005) 14 [hep-th/0501221].